S-parameter Sensitivity for Optimization of Microstrip Structures with Lumped Ports Based on Continuum Design Sensitivity

Jae-Hyeong Ko[1],Jin-Kyu Byun[2], Tae-Kyung Chung[1], and Hyeong-Seok Kim[1]
[1]School of Electrical and Electronics Eng., Chung-Ang Univ., Seoul 156-756, Korea, kimcaf2@cau.ac.kr
[2]School of Electrical Engineering, Soongsil University, Seoul 156-743, Korea, jkbyun@ssu.ac.kr

Abstract — In this paper, a generalized *S*-parameter sensitivity formula based on continuum design sensitivity is applied to optimal designs of microstrip structures with lumped ports. The practical problems that arise when the continuum design sensitivity method is implemented with commercial EM software as an analysis tool are explained and their solutions are presented. In particular, the extraction of the incident field at the lumped port and the calculation and application of the adjoint source term are described in detail. The proposed method is successfully applied to the optimization of aperture-coupled microstrip patch antenna.

I. Introduction

In recent years, there has been steadily increasing efforts to apply design sensitivity analysis (DSA) to radio frequency (RF) devices [1]-[5]. Compared to widely used stochastic optimization methods such as evolution strategy or genetic algorithm, DSA provides faster convergence in optimization problems with a large number of design parameters. Of particular interest is a generalized continuum sensitivity formula presented in [5], which is a complete sensitivity formula derived from a generalized objective function and the variational of the vector wave equation. It involves the shape changes of conducting surfaces as well as changes of interface contours between different materials. One of the advantages of this generalized DSA formulation is that it does not depend on any specific analysis method. Thus, electric and magnetic field solution from commercial EM software such as HFSS can be extracted and used in the sensitivity calculation, which greatly enhances the flexibility and applicability of the method. This means that the DSA can be applied to more practical design problems.

So far, this method has been applied to optimization of a three-dimensional dielectric resonator used in waveguide filters [4] and waveguide T-junction in three dimensions [5]. However, these numerical examples all adopted waveguide ports, and derivation and application of adjoint source required for *S*-parameter sensitivity were relatively simple. In this paper, we expand the generalized continuum sensitivity formula to the optimal design of widely used microstrip structures with lumped ports, such as microstrip patch antennas. Also, practical issues that are encountered when commercial EM software is combined with DSA are explained, and their solutions are presented. In particular, the extraction of the incident field at the lumped port and the derivation and application of adjoint source when *S*-parameter profile is given as a design goal are described in detail. The proposed method is successfully applied to the optimization of aperture-coupled microstrip patch antenna.

II. Sensitivity Calculation

In this chapter, the generalized continuum sensitivity formula from [5] is summarized. In a case of source-free medium, an objective function expressed in terms of electric fields is mathematically formulated as:

minimize

$$F = \int_\Gamma f(\mathbf{E}(\mathbf{p}))m_f \, d\Gamma \tag{1}$$

subject to

$$\nabla \times (\mu_r^{-1} \nabla \times \mathbf{E}(\mathbf{p})) - k_o^2 \varepsilon_r \mathbf{E}(\mathbf{p}) + j\omega\mu_o\sigma\mathbf{E}(\mathbf{p}) = 0 \tag{2}$$

where f is a scalar function differentiable to the electric field $\mathbf{E}(\mathbf{p})$, which is itself an implicit function of the design variable vector $\mathbf{p}$ and the characteristic function m_f in (1) denotes a part of the boundary Γ where the objective function is defined. The state variable $\mathbf{E}$ must satisfy the time-harmonic wave equation (2) which is treated as an equality constraint with homogeneous boundary conditions ($\mathbf{n}\times\mathbf{E}=0$, $\mathbf{n}\times\nabla\times\mathbf{E}=0$) on electrically/magnetically conducting surfaces. Additional boundary condition of the third kind may be also imposed on the boundary Γ, which is expressed as:

$$\mu_r^{-1} \mathbf{n} \times \nabla \times \mathbf{E} + \gamma_e \mathbf{n} \times (\mathbf{n} \times \mathbf{E}) = \mathbf{U} \tag{3}$$

where $\mathbf{n}$ is the outward normal vector, γ_e is a known parameter, and $\mathbf{U}$ is a known vector.

After applying Lagrange multiplier method, the augmented objective function $\bar{F}$ is established as:

$$\begin{aligned}\bar{F} &= \int_\Gamma f(\mathbf{E}(\mathbf{p}))m_f \, d\Gamma \\ &+ \int_\Omega [-\mu_r^{-1} \nabla \times \mathbf{E} \cdot \nabla \times \boldsymbol{\lambda} + k_o^2 \varepsilon_r \mathbf{E} \cdot \boldsymbol{\lambda} + j\omega\mu_o\sigma\mathbf{E} \cdot \boldsymbol{\lambda}] d\Omega \\ &- \int_\Gamma [\gamma_e (\mathbf{n} \times \mathbf{E}) \cdot (\mathbf{n} \times \boldsymbol{\lambda}) + \mathbf{U} \cdot \boldsymbol{\lambda}] d\Gamma\end{aligned} \tag{4}$$

where $\boldsymbol{\lambda}$ is interpreted as the adjoint variable. The variation of (4) yields the generalized sensitivity formula given by

$$\begin{aligned}\dot{\bar{F}} &= \int_\gamma [\omega^2 \mu_o (\mu_2 - \mu_1)(\mathrm{H}_{1n} \cdot \mathrm{H}_{2n}(\boldsymbol{\lambda}) - \mathrm{H}_{1t} \cdot \mathrm{H}_{2t}(\boldsymbol{\lambda})) \\ &- k_o^2 (\varepsilon_{r2} - \varepsilon_{r1})(\mathrm{E}_{1n} \cdot \lambda_{2n} - \mathrm{E}_{1t} \cdot \lambda_{2t}) \\ &- j\omega\mu_o (\sigma_2 - \sigma_1)(\mathrm{E}_{1n} \cdot \lambda_{2n} - \mathrm{E}_{1t} \cdot \lambda_{2t})] \, m_\gamma V_n d\gamma \\ &- \int_\Gamma [\gamma_e \mathrm{E}_t \lambda_t H] m_\Gamma d\Gamma\end{aligned} \tag{5}$$

where the functions, m_γ and m_Γ, denote a part of interface γ and boundary Γ where the design variables are defined, the subscripts, n and t, are the normal and tangential components to γ and Γ, respectively, H is the mean curvature and V_n is the normal component of the design velocity field $\mathbf{V}$. The subscripts, 1 and 2, in (5) mean the corresponding regions Ω_1 and Ω_2, respectively. The governing equation and its boundary conditions for the adjoint system can be found in a point form as:

$$\nabla\times(\mu_r^{-1}\nabla\times\boldsymbol{\lambda}(\mathbf{p}))-k_o^2\varepsilon_r\,\boldsymbol{\lambda}(\mathbf{p})+j\omega\mu_o\sigma\boldsymbol{\lambda}=0 \quad \text{in } \Omega \tag{6}$$

$$\begin{aligned}&\mathbf{n}\times\boldsymbol{\lambda}=0,\quad \mathbf{n}\times\nabla\times\boldsymbol{\lambda}=0\\ &\mu_r^{-1}\,\mathbf{n}\times\nabla\times\boldsymbol{\lambda}+\gamma_e\,\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\lambda})=\mathbf{f}_\mathbf{E}m_f \quad \text{in } \Gamma.\end{aligned} \tag{7}$$

where $\mathbf{f}_\mathbf{E}\equiv\partial f/\partial\mathbf{E}$.

III. Application to Microstrip Structures

For optimal design of microstrip structure with lumped ports, e.g., microstrip patch antenna, the objective function is often expressed in terms of S_{11}.

$$\text{minimize} \quad F=\frac{1}{n}\sum_{i=1}^{n}\frac{1}{2}\left[|S_{11i}|^2-|S_{11o}|^2\right]^2 \tag{8}$$

where n is total frequency points, S_{11i} is the reflection coefficient calculated at the ith frequency, S_{11o} is the desired value at each frequency. Since the adjoint source term $\partial F/\partial\mathbf{E}$ has to be calculated, $|S_{11}|^2$ is expanded in terms of the incident and total electric field, $\mathbf{E}_{inc}$ and $\mathbf{E}_{tot}$ at the lumped port as

$$|S_{11}|^2=\frac{[\mathrm{Re}(\mathbf{E}_{tot}-\mathbf{E}_{inc})]^2+[\mathrm{Im}(\mathbf{E}_{tot}-\mathbf{E}_{inc})]^2}{|\mathbf{E}_{inc}|^2} \tag{9}$$

Thus, the adjoint source term can be derived as

$$\frac{\partial F}{\partial\mathbf{E}_{tot}}=\frac{1}{n}\sum_{i=1}^{n}\left[|S_{11i}|^2-|S_{11o}|^2\right]\frac{\partial|S_{11i}|^2}{\partial\mathbf{E}_{tot}} \tag{10}$$

where $\partial|S_{11i}|^2/\partial\mathbf{E}_{tot}$ term is calculated from (9) as

$$\frac{\partial|S_{11i}|^2}{\partial\mathbf{E}_{tot}}=\frac{1}{2}\left(\frac{\partial|S_{11i}|^2}{\partial\,\mathrm{Re}(\mathbf{E}_{tot})}-j\frac{\partial|S_{11i}|^2}{\partial\,\mathrm{Im}(\mathbf{E}_{tot})}\right)=\frac{\mathbf{E}_{ref}^*}{|\mathbf{E}_{inc}|^2} \tag{11}$$

The problem that arises when implementing (11) with commercial EM software is that $\mathbf{E}_{inc}$ and reflected field $\mathbf{E}_{ref}$ at the lumped port cannot be directly extracted from the solver in most cases. Thus, they should be calculated from available field values and parameters ($\mathbf{E}_{tot}$ and S_{11}) at the port as

$$\mathbf{E}_{inc}=\frac{\mathbf{E}_{tot}}{1+S_{11}},\quad \mathbf{E}_{ref}=S_{11}\cdot\frac{\mathbf{E}_{tot}}{1+S_{11}} \tag{12}$$

Since the objective function related to S_{11} is used, the primary and the adjoint systems are self-adjoint and there is no need to solve the adjoint system. Instead, in order to obtain the adjoint field $\boldsymbol{\lambda}$, the primary field solution $\mathbf{E}_{tot}$ should be multiplied by the adjoint source term given by (10) then normalized to $\mathbf{E}_{inc}$ at the input lumped port.

IV. Optimal Design Results

Fig. 1 shows the shape of the aperture-coupled microstrip patch antenna to be optimized. The design goal is set to obtain the resonant frequency of 2.2 GHz, and the design variables are set as the location of the 4 edges of the patch as depicted in Fig. 2. Fig. 3 compares the S-parameter plots of the initial and optimized antenna. It can be seen that excellent matching is achieved after 13 iterations (S_{11} =-21.427 dB at 2.2 GHz). The change of design variables and objective function before and after the optimization is given in Table I. In the extended paper, example with more complex shape will be added and results will be compared to the case when stochastic method is used instead of DSA.

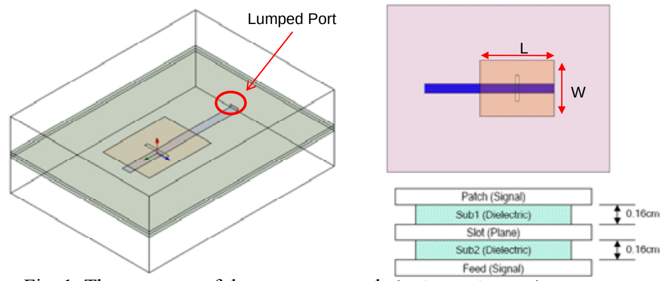

Fig. 1. The structure of the aperture-coupled microstrip patch antenna.

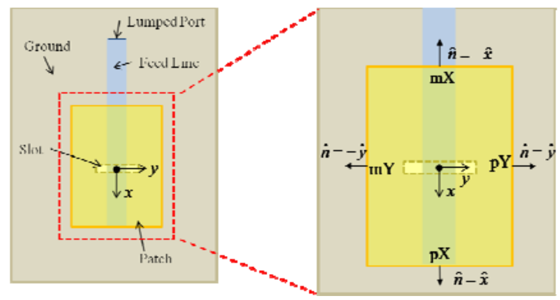

Fig. 2. The definition of 4 design variables.

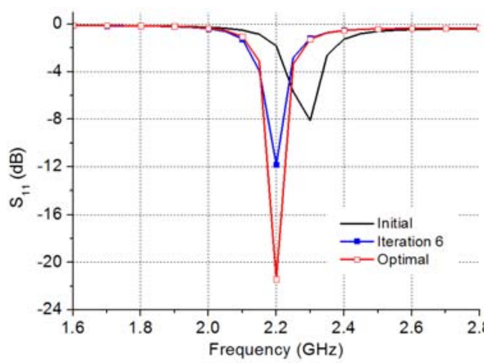

Fig. 3. Comparison of S-parameter profile.

TABLE I
Comparison of Design Variables and Objective Function

Variables	mX	pX	mY	pY	F	S_{11}
Initial	-2	2	-1.5	1.5	19.6086	-1.819
Optimal	-1.9206	2.2857	-1.6767	2.3239	0.0298	-21.427

Design variables values (cm) are the x or y coordinates of the patch boundary shown in Fig. 2. F is the calculated value of the objective function defined in (8).

V. References

[1] Jin-Kyu Byun and Il-Han Park "Design of dielectric waveguide filters using topology optimization technique," *IEEE Trans. Magn.s*, vol. 43, no. 4, pp. 1573-1576, 2007.

[2] N. K. Nikolova, et al., "S-parameter sensitivities for electromagnetic optimization based on volume field solutions," *IEEE Trans. Microwave Theory and Techniques*, vol. 57, no. 6, pp. 1526-1538, 2009.

[3] Y. Yang, T. Hallerod, et al., "Gradient optimization of microwave devices using continuum design sensitivities from the adjoint problem," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1780-11783, 2005.

[4] Dong-Hun Kim, et al., "Design optimization of waveguide filters using continuum design sensitivity analysis", *IEEE Trans Magn*, Aug. 2010 (in press).

[5] N.-S. Choi, G.-W. Jeung, J.-K. Byun, H.-G. Kim, and D.-H. Kim, "Generalized continuum sensitivity formula for shape optimization of high-frequency devices in frequency domain," *IEEE Trans. Magn.*, to be published.